$$\gamma_i = \int_0^{\alpha} \eta^i \sin[2\beta(\eta - 1 + \ell_t/\ell_p)]\eta \tag{23e}$$

$$F_i = \int_0^{\alpha} \eta^i F(\eta)\mathrm{d}\eta \tag{23f}$$

$$\alpha = (\ell_p - \ell_d)/\ell_p, \quad \beta = \ell_p/a \tag{23g}$$

The frequencies of longitudinal oscillations can now be determined by solving the eigenvalue problem obtained from Eq. (22).

Results for the Partial-Slip Case

The friction affects the natural frequencies by governing the oscillating length ℓ_p. [Note that, in Eq. (23b), $p^2r = EA/m_b\Omega^2\ell_p$ and $r = \rho\ell_p/m_b$]. The second row of Table 3 shows the oscillating length ℓ_p for a given friction coefficient μ. The table corresponds to a deployed length of 1 km and a wrapped length of 99 km. The orbital altitude is 220 km and the subsatellite mass is 450 kg. The tether parameters are as given in the no-slip case, while the diameter of the reel (i.e., $2a$) is 1 m. It is assumed that F_s is 0.1% of T_r.

It may be noted in Table 3 that, as the friction increases, the oscillating length ℓ_p reduces and the frequencies increase. Between the two extreme cases, the first natural frequency changes by an order of magnitude, while the others change by two orders of magnitude for the particular case considered. It may also be noted that, even when μ is only moderately high (0.1), the frequencies are quite close to the no-slip case.

It may also be observed from Table 3 that the lowest natural frequency can be an order of magnitude smaller compared to the rest, while the second, third, ..., natural frequencies are roughly proportional to 1,2,..., etc., respectively. As has been mentioned earlier, in the first mode, the tether behaves approximately as a spring having a mass attached to its end; in the other modes, the tether behaves as a bar vibrating axially with the end mass playing a relatively minor role.

Concluding Remarks

Longitudinal oscillation of tethered satellite systems for various levels of friction acting on the undeployed portion of the tether was studied. It was noted that, for large undeployed lengths, there can be significant differences in the frequencies for various levels of friction; however, if the friction is moderately high, the longitudinal frequencies approach those of the no-slip case, and one can use the analytical solution for that case presented in this Note in most situations.

Acknowledgment

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Dynamic Simulation of Spin-Stabilized Spacecraft with Sloshing Fluid Stores

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Introduction

LAUNCHINGS of several communications satellites have consistently resulted in a nutating motion of the spacecraft. Flight data from the roll, pitch, and yaw axis rate gyros indicated a constant-frequency, equal-amplitude, sinusoidal oscillation about the pitch and yaw axis. The vector combination of these two components of oscillation resulted in a coning motion of the satellite about the roll axis. The vehicle was spin-stabilized at launch about the minor axis, having a one revolution per second (rps) roll angular velocity imparted to it.

After launching from the carrier vehicle in the perigee phase of its orbit, the satellite's perigee assist module (PAM) fired its thruster to establish a geosynchronous Earth orbit. It is this axial thrust that gives rise to the coning that predominates after PAM motor burnout. Consistently, flight data from rate gyros indicated the steady-state one-half cycle per second (cps) coning frequency and a one-half cycle cps small-amplitude disturbance superimposed on the 1 rps roll angular velocity.

Combustion instabilities in the PAM rocket motor were suspected to be the source of a side force that would induce the coning motion. In order to investigate the presence of any such combustion instabilities, a PAM rocket motor was fired at the Engine Test Facility, Arnold Engineering Development Center, Arnold Air Force Station. A test fixture having lateral and axial load cells was utilized, allowing the PAM to be spun at 1 rps during firing. A spectral analysis was completed of the resulting load cell records obtained during firing. The test results indicated no forces at the required frequency (one-half cps), and it was concluded that combustion instabilities were not the source of moments causing coning motion.

A preliminary analysis of the payload (communication satellite) was completed indicating that sloshing fluid stores may induce the coning motion. It was suspected that sloshing motion of the liquid stores in the vehicle, excited by the axial thrust, was the mechanism for creating the nutation of the spacecraft.

The modeling of fluid slosh is extensive and has been used by researchers to study its effect on space vehicle motion.[1-3] Michelini et al.[4] outlined a procedure for developing the equations of motion of a spinning satellite containing fluid stores. The equations of motion were not presented, but the study supplied the analytical background for the experimental identification of the dynamic model. Experimental results showed that small-amplitude free surface wave motion does not cause instabilities in the vehicle. Instabilities were found to be generated by the fundamental mode of fluid slosh, which is not excited by small-amplitude free surface wave motion. The consequence of the first-mode natural frequency causing instability in the vehicle justifies the use of an equivalent spherical pendulum model of fluid slosh.

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The equivalent spherical pendulum model, which is based on experimental results, is a means of simplifying the extremely complicated problem of modeling the free surface fluid behavior coupled to the vehicle dynamics. The problem would normally consist of solving a boundary value problem and an initial value problem. The simplified equivalent mechanical model reduces the complexity to an initial value problem, which is handled with much more ease mathematically. This study uses the experimental results of previous researchers to develop a simplified mathematical model that describes the interaction between the fluid mass and main vehicle body.

Numerical Simulation of the Equations of Motion

The equations that define the motion of the system shown in Fig. 1 are a set of nonlinear coupled ordinary differential equations.[5–7] The equations must be solved by numerical methods because an analytical solution is not available.

Numerical values for the vehicle geometry and fuel tank configuration were obtained from test data. The vehicle main body mass varies from 6400 to 2000 lbm during the 85.3-s PAM burn while the roll axis moment of inertia varies from 18869 to 10240 lbm-ft^2 and the transverse axis inertia from 67652 to 19513 lbm-ft^2. Sloshing fluid stores were modeled to have a pendulous mass of 5 lbm and a length of 7.2 in. with the centers of the four tanks positioned symmetrically 25 in. along and 18 in. radially from the main body centerline. Fluid damping was estimated to have a 0.01 dimensionless damping ratio. The dimensionless damping ratio is determined from a single degree of freedom slosh analysis based on fluid type, mass, level, and tank geometry. The damping coefficient used in the model is then determined from the classical vibration relationships between the dimensionless damping ratio, pendulum geometry, and pendulum natural frequency. Symmetry of the tank geometry allows the same damping coefficient to be used with respect to both of the pendulum degrees of freedom.

The flight simulation was made with the vehicle spin-stabilized about the minor axis. Initial conditions on the vehicle were simple spin about the minor axis, with the main body fixed axis aligned with the inertial frame, an altitude of 200 miles and a 1.5-h orbital period. Figure 2 shows the body fixed angular rates vs time with instability occurring near the PAM burnout at 85.3 s. It is reasoned that the instability is caused by increasing torque on the main body resulting from the mass expulsion coupled with the fluid sloshing mass motion. Expulsion of mass not only produces time-varying inertia but also generates what may be interpreted as external torque on the main body even if the exit velocity vector is aligned with the spin axis. The torque is zero if there is no coning motion, but any small disturbance, i.e., fluid sloshing mass motion, perturbs the main body, causing it to cone more which, in turn, excites the sloshing mass motion. The torque and sloshing mass motion effects will be diminished if the main body is gyroscopically stiff enough. Before PAM burnout, the motion of the vehicle is still relatively stable because the system is gyroscopically stiff and has a transverse to roll axis inertia ratio of 3.6. After PAM burnout, the inertia is constant while the transverse to roll axis inertia ratio is reduced to 1.9 and there is a step

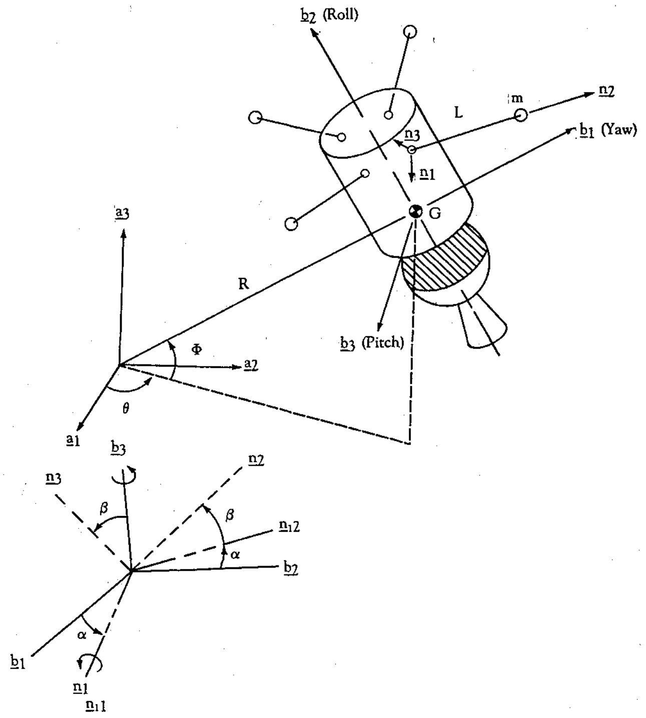

Fig. 1 Model of spacecraft with spherical pendulum.

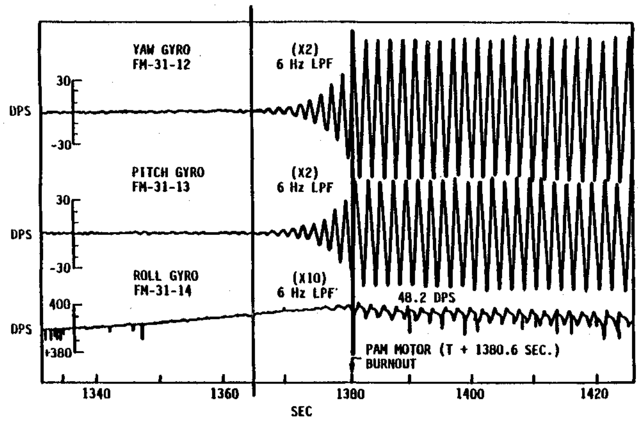

Fig. 3 Body fixed angular rates vs time—RCA-C1.

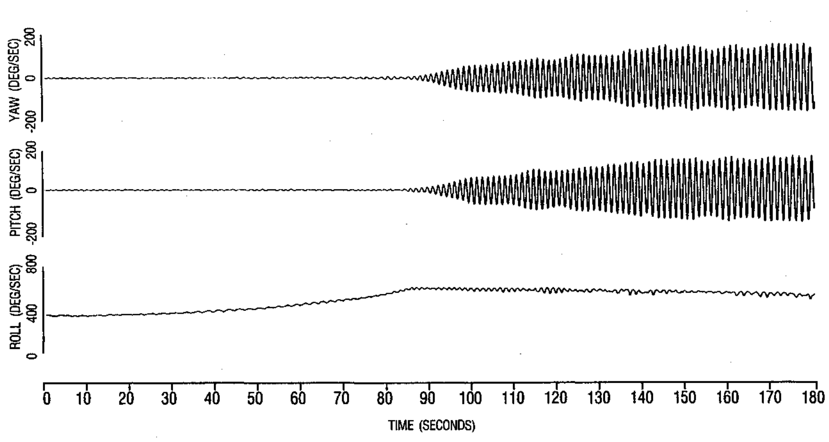

Fig. 2 Body fixed angular rates vs time with powered flight.

change in the main body acceleration. The step change in acceleration excites the sloshing mass motion which, when coupled with the main body decreased inertia ratio, produces an unstable oscillation.

Figure 3 shows telemetered flight data from a previous mission. The pitch and yaw rate gyro data show approximately equal amplitude with 90-deg phase shift, indicating a coning response. Comparison of Figs. 2 and 3 shows that the growth of the coning motion is substantially different, indicating that sloshing mass motion is not the mechanism causing the anomaly.

Conclusions and Recommendations

This study has shown that powered flight of a spacecraft carrying fluid stores within the main rigid body can be a source of dynamic instability.

The major conclusions and recommendations drawn from this study are:

1) Explicit dynamic response equations for this complex system were derived using both Kane's method and Lagrange's equation, with the fluid modeled as an equivalent spherical pendulum.

2) Sloshing fluid stores are not the source of dynamic instability seen in the launchings of STAR 48 rocket-motor-equipped spacecraft that carried the fluid stores.

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Book Announcements

NELSON, R. C., ***Flight Stability and Automatic Control,*** McGraw-Hill, New York, 1988, 284 pages.

Purpose: This book is designed for a course in airplane stability and control at the undergraduate level. It covers the basic topics of static and dynamic stability, aircraft equations of motion, aerodynamic modeling, and automatic control.

Contents: Static stability and control; aircraft equations of motion; longitudinal motion (stick fixed); lateral motion (stick fixed); response to control or atmospheric inputs; automatic control-application of conventional as well as modern control theory.

WHITFORD, R., ***Design For Air Combat,*** Jane's Publishing, London, 1987, 224 pages.

Purpose: This book is concerned with the design of the modern combat aircraft, dealing with the shapes of such aircraft and their aerodynamic rationale. It includes the design features found on the exciting breed of aircraft that appeared in the 1970's, as well as those likely to enter service in the 1990's.

Contents: Wing design; air intakes; fuselage design; tailplanes; fins; exhaust nozzles and aft-body shape.

GREENBERG, M. D., ***Advanced Engineering Mathematics,*** Prentice-Hall, Englewood Cliffs, NJ, 1988, 946 pages.

Purpose: This book is intended primarily as a text for a single- or multi-semester course in applied mathematics for students in engineering and science. It is also useful for reference.

Contents: Linear algebra; multivariable calculus and field theory; Fourier series; Fourier and Laplace transforms; partial differential equations; complex variable theory.

GREENWOOD, D. T., ***Principles of Dynamics,*** 2nd ed., Prentice-Hall, Englewood Cliffs, NJ, 1988, 552 pages.

Purpose: This book is a senior-level text in dynamics of particles and rigid bodies.

Contents: Kinematics and dynamics of a particle; dynamics of a system of particles; orbital motion; Lagrange's equation; kinematics of rigid body motion; dynamics of a rigid body; vibration theory.

THOMSON, W. T., ***Theory of Vibration With Applications,*** 3rd ed., Prentice-Hall, Englewood Cliffs, NJ, 1988, 467 pages.

Purpose: This book is a senior-level text in mechanical vibrations. It includes newer material and a modern treatment of the subject matter.

Contents: Oscillatory motion; free, harmonically excited, and transient vibration; vibrating systems; Lagrange's equation; normal modes; mode summation procedures; finite element method; approximate numerical methods; numerical procedures for lumped mass systems; random vibrations; nonlinear vibrations.

FLIESS, M. and **HAZEWINKEL, M.** (Eds.), ***Algebraic and Geometric Methods in Nonlinear Control Theory,*** D. Reidel Publishing Co., Boston, MA, 1986, 642 pages.

Purpose: This volume is the proceedings of a conference on nonlinear system theory held in Paris in June of 1985.

Contents: Controllability, observability, realization, and other structural properties; feedback synthesis and linearization techniques; optimal control; discrete-time systems; various other theoretical aspects; applications.